Problem Set 4

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Problem 1. Find the sum of the series:

$$x + \frac{x^2}{2} + \frac{x^3}{3} + ... = \sum_{n=1}^{\infty} \frac{x^n}{n}$$

Hint: derive the expanded expression, i.e. the term at the left hand side, then see if it looks familiar from class and take the integral

Problem 2. Use the formula given in the slides to find the Taylor's series of each of the following functions:

- $y = e^{-x}$
- $y = e^{3x}$
- $y = \ln(1 + x)$
- $y = \frac{1}{(1+x)}$

Problem 3. Find the domain of the following functions and express it in set notation; sketch some level curves for those functions for which is feasible:

- $f(x, y) = \frac{xy}{y-2x}$
- $f(x, y) = \frac{1}{x} + \frac{1}{y}$
- $f(x, y) = \frac{1}{(e^x+e^y)^2}$
- $f(x, y) = \ln y - 3x$
- $f(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$
- $f(x, y, z) = \frac{1}{\sqrt{16-x^2-y^2-z^2}}$

Problem 4. Find the partial derivatives w.r.t. x, y, and z; also find the cross derivatives and verify that $f_{xy} = f_{yx}$ for all combinations:

- $f(x, y, z) = x^2 y^5 z^7$
- $f(x, y, z) = x \ln \frac{y}{z}$
- $f(x, y, z) = e^{x^2+y^3+z^4}$