Problem Set 2

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Problem 1. Solve the following limits

1. $\lim_{x \to 0} \frac{x(1-x)}{3x^2}$

2. $\lim_{x \to 0} \frac{x-3}{x^2+x-12}$

3. $\lim_{x \to 1} \frac{x+2}{x^2-4}$

4. $\lim_{x \to -1} \frac{2x^2+x-6}{x+2}$

5. $\lim_{x \to \infty} \frac{2x^3-x^2+7x-3}{2-x+5x^2-4x^3}$

6. $\lim_{x \to \infty} \sqrt{x+1} - \sqrt{x}$

Problem 2. For what value of b does the function $y = x^2 + bx + 1$ have a horizontal tangent at $x = 3$?

Problem 3. Find the two points on the curve $y = x - \frac{1}{4}x^2$ at which the tangent passes through the point $\left(\frac{9}{2}, 0\right)$

Problem 4. Use the three-step rule to calculate $f'(x)$ if $f(x)$ is equal to:

1. $\frac{x+1}{x}$

2. $\frac{3-2x}{x-2}$

3. $\sqrt{3x+2}$

4. $\sqrt{x^2+1}$

Problem 5. A car riding company takes passengers from one side of the city to the other at a fix price per meter m. The costs of the company vary according to the following **Total Cost Function:** $CT(m) = 3m^2 + 5m + 2$. At what rate is the cost changing at every moment? *Hint: find the rate of change as $\Delta m \to 0$*

Problem 6. Find all points on the curve $y = \frac{6}{x}$ where the tangent is parallel to the line $2x + 3y + 1 = 0$

Problem 7. Sketch the graph of the curve $y = \frac{x}{x+1}$. how many tangent lines pass through the point $(1,3)$? Find the x-coordinates of the points of tangency of these lines.

Problem 8. Let P be a point on the first-quadrant part of the curve $y = \frac{1}{x}$. Show that the triangle determined by the x-axis, the tangent at P, and the line from P to the origin is isosceles, and find its area.

Problem 9. Find $\frac{\partial s}{\partial t}$:

1. $s = \frac{1}{(2t-1)^2}$

2. $s = \frac{t^4 - 10t^2}{(t^2 - 6)^2}$

Problem 10. Find $\frac{\partial y}{\partial x}$ by two methods, first without the power rule and then using the power rule.

1. $y = u^2, u = x^2 - 3x + 2$ 2. $y = u^3, u = x - \frac{1}{x}$

Problem 11. Find $\frac{\partial y}{\partial x}$ by implicit differentiation and also by solving for y and then differentiating, and verify that your two answers are equivalent.

1. $2x^2 + 3x + y^2 = 12$ 2. $\frac{1-y}{1+y} = x$

Problem 12. Find the tangent line of:

1. $y = (5 - 3x)^{\frac{1}{3}}$ at $(-1, 2)$ 2. $x^4 + 16y^4 = 32$ at $(2, 1)$

Problem 13. Find the first four derivatives of:

1. $8x - 3$ 3. $x^4 - 13x^3 + 5x^2 - 3x - 2$

2. $8x^2 - 11x + 2$ 4. $x^{\frac{5}{2}}$